THE MAGNETOHYDRODYNAMIC KELVIN-HELMHOLTZ INSTABILITY: A THREE-DIMENSIONAL STUDY OF NONLINEAR EVOLUTION[4]

DONGSU RYU[1], T. W. JONES[2], AND ADAM FRANK[3]
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ABSTRACT

We investigate through high resolution 3D simulations the nonlinear evolution of compressible magnetohydrodynamic flows subject to the Kelvin-Helmholtz instability. As in our earlier work we have considered periodic sections of flows that contain a thin, trans-sonic shear layer, but are otherwise uniform. The initially uniform magnetic field is parallel to the shear plane, but oblique to the flow itself. We confirm in 3D flows the conclusion from our 2D work that even apparently weak magnetic fields embedded in Kelvin-Helmholtz unstable plasma flows can be fundamentally important to nonlinear evolution of the instability. In fact, that statement is strengthened in 3D by this work, because it shows how field line bundles can be *stretched and twisted* in 3D as the quasi-2D *Cat's Eye* vortex forms out of the hydrodynamical motions. In our simulations twisting of the field may increase the maximum field strength by more than a factor of two over the 2D effect. If, by these developments, the Alfvén Mach number of flows around the Cat's Eye drops to unity or less, our simulations suggest magnetic stresses will eventually destroy the Cat's Eye and cause the plasma flow to self-organize into a relatively smooth and apparently stable flow that retains memory of the original shear. For our flow configurations the regime in 3D for such reorganization is $4 \lesssim M_{Ax} \lesssim 50$, expressed in terms of the Alfvén Mach number of the original velocity transition and the initial Alfvén speed projected to the flow plan. When the initial field is stronger than this, either the flow is linearly stable (if $M_{Ax} \lesssim 2$), or becomes stabilized by enhanced magnetic tension due to the corrugated field along the shear layer before the Cat's Eye forms (if $M_{Ax} \gtrsim 2$). For weaker fields the instability remains essentially hydrodynamic in early stages, and the Cat's Eye is destroyed by the hydrodynamic secondary instabilities of a 3D nature. Then, the flows evolve into chaotic structures that approach decaying isotropic turbulence. In this stage, there is considerable enhancement to the magnetic energy due to stretching, twisting, and turbulent amplification, which is retained long afterwards. The magnetic energy eventually catches up to the kinetic energy, and the nature of flows become magnetohydrodynamic. Decay of the magnetohydrodynamic turbulence is enhanced by dissipation accompanying magnetic reconnection. Hence, in 3D as in 2D, very weak fields do not modify substantially the character of the flow evolution, but do increase global dissipation rates.

Subject headings: instabilities – methods: numerical –MHD – plasma – turbulence

1. INTRODUCTION

Strongly sheared boundary flows are ubiquitous in astrophysical environments as diverse as the earth's magnetopause and supersonic jets. The susceptibility of such boundaries to the Kelvin-Helmholtz (K-H) instability is well-known (*e.g.*, Chandrasekhar 1961). Development of the instability may lead to turbulence, momentum and energy transport, dissipation and mixing of fluids (see, *e.g.*, Maslowe 1985 for a review).

Most astrophysical environments are electrically conducting, so relevant fluids are likely to be magnetized on length and time scales of common interest. Thus, it is important to understand the role of magnetic fields in the K-H instability. The basic linear stability analysis of the magnetohydrodynamic (MHD) K-H instability was carried out long ago (*e.g.*, Chandrasekhar 1961; Miura & Prichett 1982). There is now also a growing literature of the nonlinear evolution of the MHD K-H instability beginning from a variety of possible initial flow configurations, at least in the earlier evolution stages in two dimensions (2D) (*e.g.*, Tajima & Leboeuf 1980; Wang & Robertson 1984; Miura 1984, 1987, 1997; Wu 1986; Dahlburg *et al.* 1997; Keppens *et al.* 1999; Keller & Lysak 1999). Fully three dimensional (3D) nonlinear studies are still quite limited and so far have generally not followed flow evolution to anything resembling a final state. They do show that full coupling of magnetic field and flow in the third dimension may quickly introduce obvious dynamical effects, however (*e.g.*, Galinsky & Sonnerup 1994; Keppens & Tóth 1999; Keller, Lysak & Song 1999). The 3D simulations reported below, on the other hand, were continued over many dynamical time scales, so that the ultimate relaxed states for the flows are clear.

Strong magnetic fields, through their tension, are well known to stabilize the K-H instability. However, the considerable potential for much weaker fields to modify the nonlinear instability, and, in particular, to reorganize the subsequent flow, has only recently been emphasized. Malagoli *et al.* (1996), Frank *et al.* (1996), Jones *et al.* (1997) and Jeong *et al.* (2000) have carried out high resolution 2D and $2\frac{1}{2}$D numerical MHD simulations of the full nonlinear evolution of the K-H instability for periodic sections of 2D flows. They have demonstrated clearly

[1] Department of Astronomy & Space Science, Chungnam National University, Daejeon 305-764, Korea: ryu@canopus.chungnam.ac.kr
[2] Department of Astronomy, University of Minnesota, Minneapolis, MN 55455: twj@msi.umn.edu
[3] Department of Physics and Astronomy, University of Rochester, Rochester NY 14627: afrank@alethea.pas.rochester.edu
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that an initially weak magnetic field can fundamentally alter evolution of the K-H instability, either by disrupting the 2D hydrodynamical (HD) flow character or by enhancing dissipation during the nonlinear evolution of this instability.

While specific issues may surely depend on matching details of a simulated configuration to the physical situation imagined, the most basic insights often come from very simple, idealized model flows. In our review here we follow Frank *et al.* (1996) and Jones *et al.* (1997) in considering periodic sections of flows. Let us define the 2D computational plane as the $x-y$ plane, so that for now there is an assumed invariance along the z direction. Initially, two uniform, but oppositely directed, velocity fields along the x–direction are separated by a thin, smooth trans-sonic shear layer ($M_0 = U_0/c_s = 1$, with U_0 the velocity difference across the shear layer). The magnetic field, which is initially aligned within the $x-z$ plane, is uniform, and so is density. The most important parameter in predicting the outcome of the subsequent MHD K-H instability is the following Alfvénic Mach number of the velocity transition; namely, $M_{Ax} = U_0/c_{Ax}$, with $c_{Ax} = B_x/\sqrt{\rho}$ the projected Alfvén speed, and B_x the magnetic field component aligned with the flow in units giving magnetic pressure, $p_b = B^2/2$. We note that Jones *et al.* (1997) demonstrated when $M_{Ax} \gtrsim 2$ (for which the instability is not stabilized by magnetic tension) and the initial magnetic field is uniform that the existence of a finite B_z is largely irrelevant to evolution of the K-H instability; that is, the orientation of the field in the $x-z$ plane does not, by itself, matter, except through its isotropic pressure.

When $M_{Ax} \lesssim 4$ but with the initial magnetic field aligned with the flow, the field is a little too weak to provide linear stability ($M_{Ax} \gtrsim 2$). Modest growth of corrugations along the perturbed shear layer generates sufficient magnetic tension to prevent further development of nonlinear evolution. That is, the flow is *nonlinearly stabilized*. However, when $M_{Ax} \gtrsim 4$, the magnetic field is too weak to have much, if any, apparent influence during the linear and early 2D nonlinear phases of the K-H instability. Thus, the initial development is largely HD. So, a *Kelvin's Cat's Eye* vortex forms with its axis in the shear layer, but perpendicular to the initial flow. In 2D HD, this structure is stable. Jones *et al.* (1997), therefore, chose $M_{Ax} \sim 4$ as a convenient boundary between *strong* and *weak* magnetic field behaviors in the 2D MHD K-H instability.

Within the weak field regime, it is also possible to distinguish further two qualitatively different evolutions. Unless an initially uniform field is amplified sufficiently during one rotation of the Cat's Eye to reduce M_{Ax} to values of order unity along the vortex perimeter, magnetic stresses have little immediate dynamical influence on vortex evolution. Then, the magnetic field primarily serves to enhance dissipation of kinetic energy through expulsion of magnetic flux in the $x-y$ plane (a.k.a. flux annihilation, via tearing mode reconnection). This case of *very weak field* was called *dissipative* in Jones *et al.* (1997). In the discussion below we will label flows initiated in this regime by **VWF**. The more interesting regime is that where the initial field is too weak to prevent formation of the Cat's Eye, but strong enough that $M_{Ax} \sim 1$ at some locations within the Cat's Eye by the end of a single vortex rotation. Under those circumstances, relaxation of magnetic stresses during reconnection deforms and then disrupts the Cat's Eye. This was called the *weak field* regime or the *disruptive* regime in Jones *et al.* (1997). We will label these cases below as **WF**.

In a 2D flow, a magnetic field line is stretched by about an order of magnitude while becoming wrapped around a vortex that it once spanned. That reduces M_{Ax} by a similar factor, since the field strength increases proportionally to the length of a flux tube (*e.g.,* Gregori *et al.* 2000). Thus, it turned out in 2D $M_{Ax} \sim 20$ is the boundary between the two cases. However, even for $M_{Ax} > 20$, there can be gradual disruption of the Cat's Eye at late epoch by an accumulation of small effects from Maxwell stresses. So, this dividing line is not distinct.

In the disruptive, **WF** case, there is also a dynamical alignment between the magnetic and velocity fields during reconnection along the perimeter of the Cat's Eye, and local cross helicity ($|v \cdot B|$) is maximized. In this configuration the 2D flow returns to a laminar form, but is now stable to perturbations smaller than the size of the computational box. Jones *et al.* (1997) emphasized that 2D vortex disruption was magnetically driven, despite the fact that the initial $\beta = p_g/p_b >> 1$, where p_g is the thermal gas pressure. We note this, since it is very common to ignore dynamical influences from magnetic fields under the condition $\beta = p_g/p_b >> 1$. That measures only the relative influences of pressure gradients, not the full Maxwell stresses. The Alfvén Mach number, on the other hand, compares more closely Maxwell to Reynolds stresses, so should provide a more direct measure of the immediate dynamical consequences of the magnetic field in nonequilibrium flows. Certainly, that is the case here.

Our objective now is to extend those previous 2D results into fully 3D flows. This step is important, since it is already well established that the Cat's Eye structure so prominent and stable in plane-symmetric, 2D flows coming from the HD K-H instability is unstable to perturbations along its axis in 3D (Hussain 1984; Bayly 1986; Craik & Criminale 1986). The resultant HD flow becomes turbulent (*e.g.,* Maslowe 1985). So, we should ask how a weak magnetic field will modify that outcome. In addition, since both vorticity and magnetic flux is subject to stretching in 3D but not in 2D, and, since vortex stretching profoundly changes 3D flows when compared to those in 2D, we might expect to find that as soon as the flow evolution deviates from 2D character the characterizations listed above no longer apply. We will find, in fact, that they do still apply, but the domain of initial magnetic field strengths that can significantly influence the flow evolution is extended to weaker fields in 3D. We will also see that the morphologies and statistical properties of magnetic and flow structures expected during 3D nonlinear flow evolution depend on the strength of the initial magnetic field. A preliminary report on some of these calculations is contained in Jones *et al.* (1999), which also includes some useful animations on a CD ROM. Those same animations are presently posted on the web site: http://www.msi.umn.edu/∼twj/research/mhdkh3d/nap98.html. The plan of the present paper is as follows: In §2 we will summarize the problem set-up and numerical method. §3 contains detailed discussions of results. A brief summary and conclusion follow in §4.

2. THE PROBLEM

The simulations reported here are direct extensions of the Jones *et al.* (1997) study to fully 3D flows. The equations we solve numerically are those of ideal compressible MHD, where the displacement current and the separation between ions and electrons are neglected as well as the effects of viscosity, electrical resistivity and thermal conductivity. In conservative form, the equations are

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0, \qquad (1)$$

$$\frac{\partial (\rho \vec{v})}{\partial t} + \nabla_j \cdot \left(\rho \vec{v} v_j - \vec{B} B_j \right) + \nabla \left(p + \frac{1}{2} B^2 \right) = 0, \qquad (2)$$

$$\frac{\partial E}{\partial t} + \vec{\nabla} \cdot \left[\left(E + p + \frac{1}{2} B^2 \right) \vec{v} - \left(\vec{v} \cdot \vec{B} \right) \vec{B} \right] = 0, \qquad (3)$$

$$\frac{\partial \vec{B}}{\partial t} + \nabla_j \cdot \left(\vec{B} v_j - \vec{v} B_j \right) = 0, \qquad (4)$$

along with the constraint $\vec{\nabla} \cdot \vec{B} = 0$ imposed to account for the absence of magnetic monopoles (*e.g.*, Priest 1984). Gas gas pressure is given by

$$p = (\gamma - 1) \left(E - \frac{1}{2} \rho v^2 - \frac{1}{2} B^2 \right). \qquad (5)$$

Standard symbols are used for common quantities. The magnetic pressure is $p_b = B^2/2$ and the Alfvén speed is $c_A = B/\sqrt{\rho}$.

The simulations have been carried out in a cubic computational box of length $L_x = L_y = L_z = L = 1$. Boundaries are periodic in the directions contained within the shear layer (namely, x and z) and reflecting above and below the shear layer (namely, y). As before we have simulated flows that are initially uniform except for a hyperbolic tangent velocity shear layer in the y-coordinate, given as

$$\vec{v}_0 = -\frac{U_0}{2} \tanh \left(\frac{y - L_y/2}{a} \right) \hat{x} \qquad (6)$$

with $a = L/25$. The equilibrium flow is directed in the $-x$ direction for $y > 0.5$, and $+x$ direction for $y < 0.5$. The velocity difference across the shear layer is unity, $U_0 = 1$. The sonic Mach number of the transition is unity, $M_s = 1$, and the adiabatic index, $\gamma = 5/3$. With this configuration K-H unstable modes will have zero phase velocities in the computational reference frame. Whereas an initial B_z has no appreciable influence on $2\frac{1}{2}$D flows, because field lines could not be stretched in that dimension, we expect field line stretching in this dimension to be important in 3D. Thus, the initial magnetic field is oblique to the flow direction, with $\theta = 30°$, but parallel to the shear plane with strengths corresponding to $M_{Ax} = 2.5,\ 5,\ 14.3,\ 50,\ 143,\ 500,$ and 1.43×10^3. For comparison, $\beta = (2/\gamma)(M_{Ax} \cos\theta°/M_s)^2$ here. See Table 1 for further details. A random perturbation of small amplitude has been added to the velocity to initiate the instability.

All cases have been simulated with grids having 64^3 and 128^3 zones (labeled 'l' standing for 'low' and 'm' standing for 'medium', respectively in Table 1) to explore basic properties including resolution issues. For three representative cases with $M_{Ax} = 14.3,\ 50,\ 1.43 \times 10^3$, the calculations have been repeated again with 256^3 zones (labeled 'h' standing for 'high' in Table 1).

Each simulation has been run up to time $t = 20 \sim 50$ (see Figs. 3 and 8 for the end time). For comparison, the sound crossing time for the box is unity. With the initial perturbation applied, the Cat's Eye forms by about $t \approx 6$ in those cases where it develops. The nominal subsequent turnover time for the Cat's Eye is also $t = t_e \sim 6$. We note for reference that our computing time units here are 2.51 longer than those in Frank *et al.* (1996) and Jones *et al.* (1997), since $L = 2.51$ was set to there to match earlier papers, but they are same as those in Jeong *et al.* (2000). To aid comparison we mention that in our present units the growth times associated with the modes having wavelength $\lambda = L$, the box size, would typically be $t_g \sim 0.6-0.7$. Thus, our simulations extend typically $\sim 35-70$ K-H linear growth times for such modes. Here, we have applied random velocity perturbations to the initial equilibrium in the present simulations. Hence, modes with shorter wavelengths develop first if unstable. However, they generally merge on time scales comparable to the growth time of the mode with wavelength which is the sum of the wavelengths of merged modes. So the mentioned growth time is still a reasonable estimate of the time required for instabilities to become a significant influence on the flow.

The ideal MHD equations have been solved using a multi-dimensional MHD code based on the explicit, finite difference "Total Variation Diminishing" or "TVD" scheme. That method is an MHD extension of the second-order finite-difference, upwinded, conservative gasdynamics scheme of Harten (1983), as described by Ryu & Jones (1995). The multi-dimensional version of the code, along with a description of various one and 2D flow tests is contained in Ryu, Jones, & Frank (1995). This version of the code contains a fast Fourier transform-based "flux cleaning" routine that maintains the $\vec{\nabla} \cdot \vec{B} = 0$ condition at each time step within machine accuracy.

3. RESULTS

The seven cases listed in Table 1 include examples that in 2D exhibit "dissipative", "disruptive" and "nonlinear stabilizing" behaviors, using the terminology defined in §1. Cases that are linearly stable were not considered, since we expect no new behaviors. Cases 1-4, with $50 \le M_{Ax} \le 1.43 \times 10^3$, would in 2D have been *dissipative* in character, Cases 5-6, with $5 \le M_{Ax} \le 14.3$, would have been *disruptive*, while Case 7, with $M_{Ax} = 2.5$, would have been *nonlinearly stabilizing*.

Initially all the cases evolve in ways consistent with the 2D description in §1 for the same field strength. For Case 7 that is basically the end of the story, since the flow is nonlinearly stabilized and remains laminar through the entire simulation. Just as in the 2D simulation described by Frank *et al.* (1996) the final result is a broadened shear layer that has been subjected to very minimal kinetic energy dissipation.

In all the remaining cases a Cat's Eye develops and is subsequently destroyed. The Cat's Eye is a 2D structure, so up to the point of its formation all the flows are still quasi-2D. Recall that in 2D the Cat's Eye remained stable in the very weak field, **VWF** case, since Maxwell stresses were not built up enough to disrupt it. But in 3D this structure is HD unstable (see §3.1). However, by and large, we still find the flow characters of *dissipative* (for the **WF** case) and *disruptive* (for the **VWF** case) carry over into 3D flows, as described in detail in §3.1 and 3.2 below. One interesting deviation is that the 3D magnetic field in Case 4, with $M_{Ax} = 50$, is significantly more disruptive than it would be in 2D. Case 4 would have been defined as *dissipative* in 2D, and it exhibits similar properties in our low and medium resolution 3D simulations. On the other hand, in higher resolution 2D simulations it showed some evidence for long-term disruptive magnetic field influence through accumulated small flow distortions. Those tendencies are much more consequential in 3D, so, we will describe the in detail the 3D behaviors observed in that case in §3.3.

3.1. *Very Weak Field (**VWF**) Cases: Turbulence*

Since they come close to HD behaviors, and thus offer a useful benchmark, we begin with discussion of Cases 1, 2 and 3 ($M_{Ax} = 1.43\times10^3,\ 500,\ 143$), which would all fall under the **VWF** or *dissipative* descriptions in 2D. Fig. 1 shows at two times the spatial distributions of magnetic field strength ($|B|$) and vorticity magnitude ($|\omega|$) for Case 1ℓ and Case 1m, while Fig. 2 shows at three times the same information for the analogous high resolution, Case 1h. One can see that the global behaviors are qualitatively consistent in all three simulations. Quantitatively, the comparisons are quite similar to what we described previously in Frank *et al.* (1996) and in Jones *et al.* (1997). That is, as expected, simulations with higher resolution capture finer structures.

In 3D HD flow the Cat's Eye violently breaks up within approximately one eddy turnover, and the flow becomes highly disordered, with very little evidence of the initial shear field; *i.e.,* decaying isotropic turbulence develops (*e.g.,* Maslowe 1985). For the **VWF** MHD cases we studied (Cases 1, 2, and 3) this behavior is also seen. We can identify the root causes for the disruption of the Cat's Eye as follows: First, Hussain (1984) pointed out the importance of the growth of coherent vortex tubes that span the Cat's Eye. These features, called *rib vortices* by Hussain, are clearly present in the early snapshots of Figs. 1 and 2. Hussain pointed out that the ribs are anchored in saddle points within the flow at the ends of the Cat's Eye, so they are subjected to rapid and intense vortex stretching. This leads to non-axial stresses on the flows. A second effect is fluid elements caught in the Cat's Eye vortex move along non-circular, or roughly elliptical, paths, so that they feel time varying shear forces. Such fluid elements are known to be subject to the *elliptical instability*, when motion perpendicular to the elliptical path is allowed in 3D (Bayly 1986; Craik & Criminale 1986). Together these effects unstably distort the initially 2D character of the flow, so that the Cat's Eye breaks up violently in less than a single turnover. Fig. 2 shows that by $t = 6$ the interior of the Cat's Eye in Case 1h is filled with a knotted tangle of thin vortex tubes. They are the remnants of smaller size vortices, which are developed and gone through the processes of disruption and merger earlier. Fig. 2 also illustrates for Case 1h that between $t = 6$ and $t = 8$ the Cat's Eye has already been severely distorted. By $t = 20$ the entire flow pattern has broken down into an apparently isotropic distribution of vortex tubes.

At this point it is useful to look closely at the simultaneous evolution of the magnetic field. Fig. 2 shows us that the regions of strong magnetic field generally match the regions of strong vorticity (although there are vortex tubes which are the remnants of the initial vorticity in the problem or the early activity inside the Cat's Eye as described more in below, and so do not match with strong magnetic field). Note that magnetic fields in this case are essentially passive, and the flows are nearly ideal. It is well-known that under these circumstances the magnetic field and vorticity obey the same evolution equation (*e.g.,* Shu 1992). So the coincidence of strong magnetic field regions with strong vorticity region is, to a certain degree, expected. We emphasize, however, that the coincidence does not necessarily imply that the two vector fields are aligned. In fact, that is distinctly *not* the case along the rib vortices *as they first form*. Rather, magnetic field lines passing diagonally across the shear layer are initially stretched around the forming Cat's Eye and become embedded within the flow fields of the rib vortices. At first the fields in the rib vortices are simply stretched over the ribs, but, over the course of the Cat's Eye formation, those field lines become twisted around the ribs, like twisted-pair electrical wires. This effect enhances significantly the amount of stretching those lines undergo compared to their 2D analogs. Thus, the magnetic fields embedded in the rib vortices are substantially stronger than other field lines merely stretched around the Cat's Eye perimeter. That explains the coincidence of strong magnetic field regions with strong vorticity region at $t = 20$ in Fig. 2. It also enhances the role of weak magnetic fields in 3D compared to 2D, as we shall see.

As magnetic field lines become twisted around rib vortices they soon develop topologies unstable to reconnection (*e.g.,* Lysak & Song 1990), however. The reconnection preserves helicity (*e.g.,* Ruzmaikin & Akhmetiev 1994), although other topological field measures, such as twist, writhe and kink that contribute to helicity may change (*e.g.,* Berger & Field 1984; Bazdenkov & Sato 1998). The product by the time the Cat's Eye begins to be disrupted is a set of twisted magnetic flux tubes around the perimeter of the Cat's Eye that do align themselves with the original rib vortices; *i.e.,*in the $x-y$ plane.

Simultaneously, the complex motions in the Cat's Eye interior, which were mentioned above, lead to extensive magnetic reconnection events that produce a 3D version of magnetic *flux expulsion* mentioned in §1 (Weiss 1966; see Jones *et al.* 1997 for discussion of that process). The product, when the Cat's Eye begins to break up, is a region of very weak and tangled magnetic field inside the Cat's Eye that on average trends in the z direction. The total magnetic flux through the full computational box is constant, of course, but this series of events has separated the magnetic flux embedded in the Cat's Eye into relatively strong flux tubes perpendicular to the axis of the Cat's Eye and wrapped around it, plus largely disordered magnetic flux inside the Cat's Eye with a mean field aligned with the axis of the Cat's Eye. This dichotomy is retained in the magnetic field structures at the end of end of Case 4h, in fact, as we will address that in §3.3.

The evolution of energy partitioning is illustrated for the **VWF** cases along with Case 4 in Fig. 3. Keep in mind that because we use a periodic box in the x and z directions and hard walls in the y direction, the system is effectively closed and total energy is conserved. It is, therefore, a necessarily decaying dynamical system, since there is finite numerical dissipation. There is an abrupt, but almost imperceptible ($< 1\%$) decrease in kinetic energy within the flow around $t \sim 6$, caused directly by formation of the Cat's Eye. Beginning with Cat's Eye disruption, however, there is a steady, steep decay of this quantity. By the end of this simulation the kinetic energy has dropped by about two orders of magnitude or more. This is in sharp contrast to the analogous 2D version of this K-H derived flow, where after formation of the Cat's Eye the kinetic energy is virtually constant on these time scales. Turbulent decay in 3D is the reason for the difference, of course. Our result is, indeed, consistent with studies of 3D decaying MHD and HD turbulence (*e.g.,* Mac Low *et al.* 1998; Stone *et al.* 1998; Porter *et al.* 1994), which also showed rapid dissipation of kinetic energy.

Three points are noticed from the kinetic energy plot of Fig. 3. First, in the medium resolution simulations, the decay rate increases with increasing initial magnetic field, from Case 1 to Case 4. This is the the result of enhanced dissipation through reconnection in MHD turbulence. Hence, even an obviously very weak magnetic field with $M_{Ax} \gtrsim 50$ (or $\beta \gtrsim 2.25\times10^3$ in our setup) does play an important role as an agency of enhancing dissipation. This character of increasing dissipation was also observed in 2D, although the dissipation there occurred through reconnection around the stable Cat's Eye instead of reconnection driven by turbulent motion (Jones *et al.* 1997). Second, in Case 1 the decay is faster in the high resolution simulation than in the medium resolution calculation. This is because higher resolution allows a greater number of smaller scale structures to form and, so, reconnection events are more frequent. Again, a similar behavior was observed in the 2D **VWF** cases (Jones *et al.* 1997). Finally, the evolution curve of kinetic energy in Case 4 is quite different in the two simulations with different resolution. This happens because in the high resolution simulation, the magnetic field is amplified enough locally to play a more important dynamical role. The details are described in §3.3.

The magnetic energy plot of Fig. 3 shows the following behavior. Initially the magnetic energy increases at the expense of the kinetic energy, but stops increasing before energy equipartition is reached. After that, the magnetic energy starts decreasing, but the rate of decline is smaller than that of the kinetic energy. During this period, the flow character is close to that of HD turbulence. But eventually, progressing from smaller scales to larger scales (see the discussion on energy power spectrum below) the magnetic energy catches up the kinetic energy, and the character of MHD turbulence is fully established. Then, both energies decay with the same rate. The turbulence developed by the K-H instability in a closed system is a decaying, quasi-isotropic turbulence, and the symmetry of this flow does not support dynamo action. Hence, the magnetic energy must decay on some time scale along with the kinetic energy, and both energies should convert into the thermal energy. Over a very long time dependent on the effective magnetic Reynolds number at the dissipation scale (see below for more discussion), but much longer than our simulation, the magnetic field in this closed system should return to something resembling the initial configuration.

In simulations of ideal MHD flows, resistivity, η, is provided by numerical truncation and diffusion at the grid-cell level. So it does not have a constant value, but depends on the size of structures considered. In a numerical code based on a second-order scheme, such as the TVD scheme, the effective numerical resistivity is inversely proportional to the square of the scale, ℓ, $\eta \propto \ell^{-2}$ (Ryu *et al.* 1995). As a result, the effective magnetic Reynolds number is proportional to the square of the scale, $R_m \propto \ell^2$. We can use the evolution of magnetic energy in our simulations, in fact, to estimate heuristically the effective magnetic Reynolds numbers as follows. The magnetic energy decay rate for non-ideal decaying incompressible MHD turbulence (*e.g.*, Biskamp 1993) is just

$$\frac{dE_m}{dt} = -\eta\int j^2 d^3x = -\eta\int(\nabla\times B)^2 d^3x \sim -2\eta\frac{E_m}{L_3^2}, \tag{7}$$

where L_3 is the thickness of current sheet. From this we can write roughly that the magnetic field decay time is

$$t_{dm} \sim \frac{L_3^2}{2\eta} \sim \frac{R_m L_3}{2v}, \tag{8}$$

where

$$\eta \sim \frac{vL_{diss}}{R_m} \tag{9}$$

and $L_{diss} \sim L_3$ are used since L_{diss} represents the scale on which energy dissipation by reconnection occurs, that is the typical thickness of current sheets. Here, v represents the typical flow velocity across the current sheet. For the Case 1h simulation using 256^3 grid zones, for instance, we estimate from Fig.3 that $t_{dm} \sim 20$ and $\sqrt{\langle v^2\rangle} \sim \sqrt{2E_k/\rho} \sim 5\times10^{-2}$ at $30 \lesssim t \lesssim 50$. So for an effective magnetic Reynolds number corresponding to the typical scale of current sheet thickness, $L_3 \sim 10^{-2}L$ (see Fig. 6 for an estimate of L_3), we obtain $R_m \sim 200$. Note that the smallest values for the scale L_3 correspond to $2-3$ grid zones, so they are numerically limited. The inertial range of turbulence in a simulation should require $R_m \gtrsim 10^3$. Hence, applying the inverse square effective dissipation behavior of our second-order scheme, an inertial range is possible on scales greater than roughly ~ 8 zones. That is, in our simulations, turbulence can be approximately represented on scales larger than ~ 8 zones.

Additional insights about the evolution of fluid and magnetic field properties can be gleaned from the 3D power spectra of the kinetic end magnetic energies, $E_k(k)$ and $E_m(k)$, respectively, defined as follows. The Fourier amplitude of the kinetic energy is calculated as

$$A_{k,j}(\vec{k}) = \frac{1}{L_xL_yL_z}\int H_xH_yH_z\sqrt{\rho}v_j \exp\left[i(k_xx+k_yy+k_zz)\right]d^3x, \tag{10}$$

and similarly the Fourier amplitude of the magnetic energy is calculated as

$$A_{m,j}(\vec{k}) = \frac{1}{L_xL_yL_z}\int H_xH_yH_zB_j \exp\left[i(k_xx+k_yy+k_zz)\right]d^3x, \tag{11}$$

where $j \in \{x,y,z\}$. Here, H_x, H_y and H_z are the Hanning window functions (Press *et al.* 1986), which are given as

$$H_x = \frac{1}{2}\left[1-\cos\left(2\pi\frac{x}{L_x}\right)\right], \tag{12}$$

and similarly for H_y and H_z. Windowing in y is used because the flow is not periodic in that direction. Then, it seems desirable to window in x and z, too, in order avoid artificial anisotropies in Fourier space. Assuming isotropy on the scales of interest (see Fig. 5 and discussion in below), the energy power spectra are given as

$$E_k(k) = \frac{L_xL_yL_z}{2(2\pi)^3W}\left(|A_{k,x}(k)|^2+|A_{k,y}(k)|^2+|A_{k,z}(k)|^2\right)k^2, \tag{13}$$

and

$$E_m(k) = \frac{L_xL_yL_z}{2(2\pi)^3W}\left(|A_{m,x}(k)|^2+|A_{m,y}(k)|^2+|A_{m,z}(k)|^2\right)k^2, \tag{14}$$

where

$$W = \frac{1}{L_xL_yL_z}\int H_x^2H_y^2H_z^2d^3x. \tag{15}$$

Note that with the above definition

$$\int E_k(k)dk = \frac{1}{L_xL_yL_zW}\int H_x^2H_y^2H_z^2\frac{1}{2}\rho v^2d^3x, \tag{16}$$

and

$$\int E_m(k)dk = \frac{1}{L_xL_yL_zW}\int H_x^2H_y^2H_z^2\frac{1}{2}B^2d^3x, \tag{17}$$

that is, $E_k(k)$ and $E_m(k)$ are the kinetic and magnetic energies per unit k, respectively.

Fig. 4 shows the above two energy power spectra, along with their sum, $E_{k+m}(k) \equiv E_k(k)+E_m(k)$, for Case 1h at t = 6, 8, 12, 20, 32 and 50. Also, for comparison, we include lines with $E \propto k^{-5/3}$ and $E \propto k^{-3}$, representing the canonical forms for inertial range 3D and 2D isotropic HD turbulence, respectively (*e.g.,* Lesieur 1997). The vertical dotted lines indicate the scales of $L/4$, one fourth of the box size, and 8 zones ($\log k = 0.602$ and $\log k = 1.505$). Structures with $\ell \sim L$ (more specifically, $\ell \gtrsim L/4$ according to our tests) have been strongly affected by the finite box size, while structures with $\ell \lesssim 8$ zones have been severely dissipated by numerical diffusion (see above). So we may regard the region only between the two vertical dotted lines as an approximately inertial range. We have seen in Fig. 2 that at the earlier times, $t = 6$ and 8, there is still considerable large-scale, non-isotropic organization to the flows (*e.g.,* the Cat's Eye). But later, at $t = 20$, the flow looks to the human eye as though it is isotropic turbulence. Fig. 4 supports that impression. In particular, if we examine $E_{k+m}(k)$ in the inertial range, we see that the power-law slope starts with a value close to –3, as expected from the 2D flow character of the Cat's Eye. Then, over time, the $E_{k+m}(k)$ becomes flatter, but the slope is still steeper than $-5/3$, until the flow develops into something very close to decaying isotropic turbulence by $t \sim 20$. After that, the amplitude decays with time, but the form remains relatively unchanged to the end of the simulation.

Another point to emphasize is that at the early epochs $E_k(k)$ dominates $E_m(k)$ on all scales. This fact is consistent with our earlier conclusion that in the **VWF** cases, the flow is initially essentially HD in character. But, as complex flow structures develop, magnetic field is amplified by flux stretching. By $t \sim 20$, $E_m(k)$ has caught up $E_k(k)$ on small scales. By the end of the simulation, $E_m(k) \sim E_k(k)$ over the most of inertial range except on the largest scales. Hence, by this time the flow of Case 1h shows the character of MHD turbulence.

The evolution to a quasi-isotropic flow character in the **VWF** case can be seen by looking at Fig. 5. This shows for Case 1h at a sequence of times $\langle v_x(y)\rangle_{x,z}$, which is the average of v_x over the $x-z$ plane. $\int \langle v_x(y)\rangle_{x,z} dy$ is always very close to 0, from the symmetry of the initial conditions, although, since the initial perturbations were random, there is no exact symmetry in y required. The shear in v_x, represented by $d\langle v_x(y)\rangle_{x,z}/dy$, keeps decreasing rapidly. By $t = 40$, not only $d\langle v_x(y)\rangle_{x,z}/dy \approx 0$, but also $\langle v_x(y)\rangle_{x,z} \approx 0$ for all y. This indicates there is no residual shear left, and the flow has become isotropic.

There are a number of quantitative ways to characterize the structural evolution of the flows in MHD simulations. The following quantities are particularly simple and useful: The mean magnetic curvature radius, L_1,

$$L_1 \equiv \sqrt{\frac{\langle B^4\rangle}{\left\langle \left[\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B}\right]^2\right\rangle}}, \qquad (18)$$

the flow Taylor microscale, L_2,

$$L_2 \equiv \sqrt{\frac{\langle v^2\rangle}{\left\langle \left(\vec{\nabla}\times\vec{v}\right)^2\right\rangle}}, \qquad (19)$$

the magnetic Taylor microscale, L_3,

$$L_3 \equiv \sqrt{\frac{\langle B^2\rangle}{\left\langle \left(\vec{\nabla}\times\vec{B}\right)^2\right\rangle}}, \qquad (20)$$

and the magnetic intermittency I,

$$I \equiv \frac{\langle B^4\rangle}{\langle B^2\rangle^2}, \qquad (21)$$

(*e.g.,* Lesieur 1997 for L_2; Ethan T. Vishniac, private communication 1999 for others). The first of these, L_1, measures how sharply the magnetic field lines are bent. L_2 and L_3 measure the transverse dimensions, or thicknesses, of vortex tubes and current sheets, respectively. I measures spatial contrast in the magnetic field strength distribution; *i.e.,* $I >> 1$ signifies the presence of magnetic voids and relatively intense flux tubes.

Fig. 6 shows the evolution of the above quantities for the three high resolution simulations Case 1h, 4h and 5h, which are **VWF**, **VWF/WF** (transitional) and **WF** cases, respectively (see the next two subsections for discussions on Cases 4h and 5h). Initially the magnetic field is uniform, so that L_1 and L_3 are infinite, while $I = 1$. The initial shear layer gives $L_2 \approx \sqrt{3La/4} \approx 0.17$. We note that for the boundary conditions used the mean vector magnetic field, $\langle\vec{B}\rangle$, is exactly constant; *i.e.,* there is no dynamo action. So, any net increase in magnetic energy must also lead to the increase in the magnetic intermittency, $I > 1$. The rest of this paragraph focuses on Case 1h, the **VWF** case. One can see from L_1, L_3 and I that very quickly, on a time scale $t \lesssim 1-2$, the magnetic field is drawn into thin structures. Especially, reduction of L_1 signals formation of highly bent or twisted field regions. At the same time L_2 is reduced, because vortices of smaller scales form. The modest increase in L_2 just before $t \sim 4$ is due to the merger of smaller scale vortices, whose remnants are seen at $t = 6$ in Fig. 2. By $t \sim 6-8$, when the Cat's Eye is formed and begins to break up, magnetic intermittency, I, is already very large. Field curvature, L_1, stays low, but shows a peak around $t \sim 8$. This is because the field is wrapped into the Cat's Eye. However, L_1 increases due to partial field relaxation during reconnection just prior to the Cat's Eye break-up. During Cat's Eye break-up the field becomes twisted and tangled, so that L_1 is again reduced. After the Cat's Eye breaks up and the memory of the initial shear is gone in this case ($t \gtrsim 20$), there begins a gradual relaxation in all of $L_{1,2,3}$; that is, the size of vortex tubes increases and the field becomes less strongly curved, while the thickness of current sheets increases and the magnetic intermittency decreases. The last of these remains relatively steady for Case 1h, near $I \sim 2$, for $t \gtrsim 30$, while the others slowly increase to the end of that simulation. This behavior reflects the fact at late time that magnetic field is gradually relaxed by straightening itself, but flux tubes remain stable structures. These properties match the finding of the slow decay of magnetic energy for Case 1h in Fig. 3.

3.2. *Weak Field (*WF*) Cases: Magnetic Reorganization of the Cat's Eye*

This subsection discusses Cases 5 and 6 ($M_{Ax} = 14.3,\ 5$), which in 2D were categorized as the **WF** or *disruptive* cases. The character of 3D flow and magnetic field evolution in these cases is perhaps best illustrated in the morphologies of Fig. 7. It shows the spatial distributions of magnetic field strength (B) and vorticity magnitude (ω) in the high resolution simulation Case 5h at three epochs. Initially, the Cat's Eye forms in this case, so the morphologies at $t = 6$, although more sheet-like, carry some resemblance with those of Case 1h in Fig. 2. But, in Case 1h the analogous images at $t = 20$ showed a completely disordered arrangement of magnetic and vortex tubes. Here those features are clearly laid out in patterns aligned to the original flow. Furthermore, a closer examination shows both the magnetic flux tubes and the vortex tubes to have a remarkable, sheet-like morphology with their minimum extent in the y direction. Thus, they follow and resemble the original shear layer itself. There is also a good correspondence between strong vorticity regions and strong magnetic field regions, as one might expect in the presence of self-organization.

An interesting point for this simulation is that 2D cuts through fixed z's resemble very much the 2D simulations of **WF** cases (see the images in Frank *et al.* 1996 and Jones *et al.* 1997). This is the result of the sheet-like morphology, and the indication that the flow and magnetic field evolution, although all three dimensions are available to the flows, the behavior is essentially 2D in character. So the **WF** cases in 3D evolve towards some degree of self-organized shear just as in 2D; that is, the Maxwell stresses developed during formation of the Cat's Eye noticeably reorganize the flow and lead to significant alignments between magnetic and velocity fields. As for the **VWF** cases (Cases 1-3), the magnetic field itself becomes organized during Cat's Eye development through the action of rib vortices into relatively strong-field flux tubes parallel to the original velocity field, separated from relatively weak fields trending along the Cat's Eye axis. Subsequently, the reorganized velocity field aligns with the stronger magnetic field and retains a clear *memory* of the original velocity shear. Again, that contrasts with the vector fields in the weakest field **VWF** or HD cases in 3D, which become essentially isotropic in nature, excepting that the mean vector magnetic field must remain unchanged, due to the symmetry.

The above point is obvious in Fig. 8, which shows at a sequence of times $\langle v_x(y)\rangle_{x,z}$ for the **WF** cases as well as the nonlinearly stable case (Case 7). The bottom panel in Fig. 8 shows at $t = 30$ the shear strength, $d\,\langle v_x(y)\rangle_{x,z}/dy$, in the original midplane of the shear layer. Two points are made from the figure. First, not only in the nonlinearly stable case but also in the **WF** cases, there left is still a well defined shear layer which is also laminar. This is the result of reorganization. Second, the residual shear strength at this time clearly scales with the initial magnetic field strength (or more importantly with B_{x0}). In HD, linear shear is stable against linear perturbations but unstable to 3D finite-amplitude perturbations (Bayly *et al.* 1988). But the magnetic field has the stabilizing effects, just as in the MHD K-H instability case. Stronger field can stabilize flows with larger linear shear. The linear correlation of the residual shear with the initial field strength is the direct consequence.

Among other things the self-organization and associated laminarity in the **WF** cases substantially slow the rate of kinetic energy dissipation, since it reduces the energy transfer to small, dissipation scales. That point is clearly made by comparing Fig. 3 and Fig. 9, which illustrate the evolution of energy partitioning for the **VWF**, **VWF/WF** (transitional) and **WF** cases. We make three points from Fig. 9. First, there is less kinetic energy dissipation in the stronger field Cases 6 than in Case 5, as expected from the above discussion on the residual shear. Second, in both **WF** Cases 5-6, about half of the initial kinetic energy is still present at $t = 20$, and the decay rate has reduced significantly from what it was during the time of Cat's Eye disruption. So, this flow pattern should continue for a moderately long time, but not as long as we found in 2D, since small scale structures in the third dimension can still form and enhance the dissipation. Third, a comparison of Cases 5h and 5m shows a good match between them. This indicates that small scale structures do not play a major role, although they do exist. At the same time, by this measure, we can state safely that the simulations are reasonably well resolved in the **WF** cases.

The overall behaviors of the mean magnetic curvature radius, L_1, the flow Taylor microscale, L_2, the magnetic Taylor microscale, L_3, and the magnetic intermittency I in Case 5h are similar as those in Case 1h, as seen in Fig. 6. Three differences are noticed. First, the L's remain small in Case 1h since there is little dynamical self-organization, But in Case 5h, self-organization relaxes the magnetic field as well as vortices. As a result, L's increase after the Cat's Eye starts to break apart. Second, the small peak in L_2 around $t \sim 4$ is missing in the **WF** case. This is because initially the formation of smaller scale vortices inside the Cat's Eye is not allowed due to the magnetic field, although weak. This agrees with the visual impression that structures are absent inside the Cat's Eye at $t = 6$ in Fig. 7. Finally, I approaches unity after $t \gtrsim 20$, indicating the magnetic field has returned, more or less, to the initial uniform configuration.

3.3. *Case 4: A Transitional* **VWF/WF** *Case with Eventual Reorganization*

Case 4 with $M_{Ax} = 50$ begins with a magnetic field too weak in 2D to have any immediate direct dynamical role, although through an accumulation of small magnetic field-induced perturbations even the 2D version of this case eventually begins to be distorted. Thus, in 2D we would have classified this as a dissipative case with a "footnote". Here we will use the label **VWF/WF**. On the face of it, the 3D Case 4 looks during formation of the Cat's Eye like the **VWF** cases discussed earlier, resembling the morphologies in Figs. 1 and 2 at $t = 6$. There is even a briefly chaotic flow pattern right after the Cat's Eye breaks up. However, in the high resolution simulation, Case 4h, slowly, over time, the flow begins to reorganize, so that by the end of the simulation residual shear becomes dominant, while the magnetic field has organized into one predominant flux tube parallel to the flow. This behavior is clearly visible in Fig. 10. So, effectively, this case behaves like the **WF**, disruptive cases discussed in the immediately preceding subsection. This case provides an evidence that the range of dynamically influential magnetic fields is greater in 3D than in 2D.

The causes of that difference can be seen clearly by a closer examination of magnetic field evolution during formation of the Cat's Eye. The key is evident in Fig. 11, which at $t = 8$ shows regions where the Alfvén Mach number is less than unity in Case 4h. This is just as the Cat's Eye begins to fall apart. The regions with $M_A < 1$ are all coincident with rib vortices that initially were HD in character. Now, however, they are magnetically dominated. Within the rib vortices at this time the flows are sub-Alfvénic, with mostly $0.1 \lesssim M_A < 1$. An image showing the regions with small $\beta = p_g/p_b$ would be almost identical in appearance to Fig. 11, as well. The smallest values of $\beta \gtrsim 1$, so it is really the tension force rather than the pressure force that is revealing the magnetic field's role.

In 2D we would have expected M_A to drop by about one order of magnitude from its initial value, since the magnetic field lines around the vortex perimeter are stretched by about that much due to formation and rotation of the Cat's Eye. That is insufficient to produce the properties seen in Fig. 11 and consistent with the observation of Jones *et al.* (1997) that a 2D $M_{Ax} \sim 50$ flow would not lead to magnetic dominance. In 3D, however, formation of the rib vortices provides a new mechanism to enhanced field amplification, as mentioned earlier. In particular, field lines become wrapped around the rib vortices, so that they become twisted as well as stretched around the Cat's Eye. Fig. 11 shows this effect by tracing field lines within one rib vortex. Those field lines are clearly twisted around the structure, so that this feature is a legitimate flux tube with sufficient magnetic tension to begin a self-organization of the flow field. A close examination of the magnetic field distribution at this time reveals strengths about ten times amplified over the initial field as expected; namely, *i.e.*, $|B| \gtrsim 0.2$ along the entire length of each of the flux tubes visible in Fig. 11. But, each tube also contains a core down much of its length that has $|B| \gtrsim 0.4$, an additional enhancement we attribute to twisting. The fraction of the flow under magnetic control at this time is still small, so the influence is not immediately obvious. It is, however, crucial to the eventual character of the flow. Evidently, if field amplification in any significant region is able to reduce the Alfvén Mach number to less than unity before the Cat's Eye is HD disrupted, some memory of the original shear will be retained and, through self-organization, the magnetic and flow fields will align, and the flow may be smoothed. The evolution of these features for Cases 4h and 5h is clearly seen in animations of vorticity and magnetic pressure published in the CD ROM along with Jones *et al.* (1999), and currently posted at the web site given at the end of §1.

The ways in which the above physics impacts on energy evolution are shown for this case in Fig. 3. We see that in the high resolution simulation energy dissipation is intermediate between the quasi-HD Case 1h, which became turbulent, and Case 5h (in Fig. 9), which quickly developed into a smooth flow. We have noted previously that in medium and low resolution simulations Case 4 behaves as a **VWF** flow, since then numerical dissipation prohibits enough amplification of magnetic field to allow it to dominate dynamics. In addition, we can see that in Case 4h the rate of kinetic energy decay drops significantly after $t \sim 25$. By that time the flow has begun to organize strongly, and initially numerous magnetic flux tubes, twisted by vortical motion, have merged into a single, relatively intense structure. Note in this respect from Fig. 11 that the magnetic energy, E_m, in Fig. 3 is relatively constant from that time on, as well.

Fig. 12 shows images of the magnetic flux structures of Case 4h at $t = 40$. The dominant flux tube is obvious. It contains most of the original flux that passed through the $x = 0$ and $x = L_x$ faces of the computational box. Originally all the field lines ran obliquely in the $x-z$ plane, but now most of the magnetic energy is concentrated in this one structure, aligned in the x direction alone. On the other hand, for the periodic boundary conditions applied here the magnetic flux through each of the individual faces of the computational box is preserved. So, there must have been a topological change in the magnetic field along the way. That fact can also be seen in Fig. 12, where we see most of the flux through the $z = 0$ and $z = L_z$ faces is now provided by field lines that meander around this dominant flux tube, and essentially orthogonal to it. That is, the magnetic flux has separated as a result of the instability and self-organization into two distinct domains.

Examination of the various structure measures in Fig. 6 augments the sense that Case 4h represents a transition between the quasi-HD Case 1 and the immediately reorganized Case 5. The magnetic field curvature measure, L_1, and the magnetic Taylor microscale, L_3, evolve in very similar ways for Cases 4 and 5, reflecting the fact that the magnetic field in each case is strong enough to smooth the flow and coalesce into one dynamically important flux tube. On the other hand, there is a much closer match between Cases 1 and 4 with regard to the flow Taylor microscale, L_2, reflecting the development of chaotic flow in each of these cases. The magnetic intermittency, I, stays moderately large, approaching $\sim 2-3$ by the end of the simulation, confirming the formation of one big flux tube.

4. SUMMARY AND DISCUSSION

Through high resolution MHD simulations using up to 256^3 grid zones, we have studied the 3D nonlinear evolution of the compressible MHD K-H instability. As in our earlier work, we have considered periodic sections of flows that contain a thin shear layer, but are otherwise uniform. The initially uniform magnetic field is parallel to the shear plane, but 30° oblique to the flow itself. Its strength spans the range corresponding to $M_x = 2.5 \sim 1.43 \times 10^3$. The sonic Mach number of the flow transition was initially unity.

The most important consequence of this work is confirmation in 3D flows of the conclusion from our earlier 2D work (Frank *et al.* 1995; Jones *et al.* 1997; Jeong *et al.* 2000) that *even apparently weak magnetic fields corresponding to* $M_{Ax} \gtrsim 4$ *can be important to nonlinear evolution of the K-H instability*. The role of weak magnetic fields has been manifested in the following two ways. First, in the case of *very weak field* (**VWF** or *dissipative* case) with $M_{Ax} \gtrsim 50$, dissipation is enhanced through magnetic reconnection. In this case, the instability remains essentially HD in character to the end. That is, the Cat's Eye is destroyed by HD secondary instabilities and the flows are developed into mostly isotropic turbulence. But the decay rate of the turbulence increases. Second, in the case of *weak field* (**WF** or *disruptive* case) with $4 \lesssim M_{Ax} \lesssim 50$, the Cat's Eye is destroyed by the magnetic stress of the field which has been amplified by stretching and twisting on the perimeter of the vortex, once the Alfvén Mach number of flows around the Cat's Eye drops to unity or less. The flows, in this case, are eventually self-organized into relatively smooth ones with linear shear, which are stable against further instabilities.

There are two noticeable differences in the results of the current 3D work from that of our previous 2D work. First, in the **VWF** cases, while the Cat's Eye remains stable in 2D, it is destroyed in 3D by inherently 3D instabilities (Hussain 1984; Bayly 1986; Craik & Criminale 1986). This is a HD process, which was fully studied before. The second difference, a 3D MHD process, is additional amplification of magnetic field by *twisting* inside rib vortices developed hydrodynamically around the Cat's Eye. That leads to an increased role for magnetic stresses in destroying the Cat's Eye. In both 2D and 3D **WF** cases magnetic field caught in the initially quasi-HD roll-up of the Cat's Eye vortex tubes within the initial shear layer is amplified by field-line *stretching*. In 2D the field strength on the perimeter of the vortex is increased by about an order of magnitude, representing the increased length of field lines dragged around the forming Cat's Eye, before they become subject to magnetic reconnection. In 3D that effect is further enhanced by the development of rib vortices spanning the Cat's Eye that twist field lines into flux tubes, which then span the Cat's Eye and apply a tension force to the plasma. In our 3D simulation, twisting of the field increases the maximum field strength by more than a factor of two over the 2D effect. For our rather idealized uniform density and vector field configurations, the boundary field strength for the **WF** case decreases to the value corresponding to $M_{Ax} \sim 50$ in 3D from $M_{Ax} \sim 20$ in 2D.

Two additional interesting points can also be made. First, in the **WF** cases, the magnetic energy reaches its maximum just as magnetic stresses begin to destroy the Cat's Eye. However, it returns very close to its initial value, becoming almost uniform magnetic field again, on the time scale same as the flow becomes organized. The latter is barely longer than the time needed to form the Cat's Eye and disrupt it. Hence, the field effectively plays the role of a *catalyst*. Second, in the **VWF** cases, where the flows become turbulent and, so, are not reorganized, the amplified magnetic field during the development of turbulence is retained long afterwards. The magnetic energy decays slowly, until it catches up to the kinetic energy. Then the flows approach decaying MHD turbulence, so that the magnetic energy along with the turbulent kinetic energy decays at an enhanced rate.

In the cases of strong field with $M_{Ax} \lesssim 4$, the development of the MHD K-H instability is essentially 2D in character, even though variation along the third dimension is allowed. When $M_{Ax} \lesssim 2$, the MHD flow is linearly stable and the instability is not initiated. When $2 \lesssim M_{Ax} \lesssim 4$, the shear layer is initially corrugated, but the enhanced magnetic tension due to the corrugated magnetic field stabilizes the instability before the Cat's Eye forms; that is, the flow is nonlinearly stable.

The model configurations studied have no global helicity, and, thus, are not capable of dynamo action. Indeed the mean vector field is a constant throughout the simulations. So, the enhancement of magnetic energy comes from twisting followed by stretching of magnetic field lines, and/or from maintenance of significant magnetic intermittency (non-uniformity), not through generation of a large-scale field. We have seen that in the transitional **VWF/WF** case, however, the enhancement of magnetic energy is maintained through concentration of magnetic flux in flux tubes. Beyond the dynamical impact of such flux concentrations, that tendency could also be significant astrophysically for a different reason. In particular, measures of magnetic field strength, such as Faraday rotation, Zeeman splitting and the intensity of synchrotron emission become biased to those localized structures, so it becomes important to establish the intermittency of the field to understand associated observations.

We note that our simulations have been done in an idealized box with periodic boundaries along the x and z-directions and reflecting boundaries along the y-direction. As a result, they have the following practical limitations. First, some astrophysical systems subject to the K-H instability, such as jets, contain continuous supplies of kinetic energy, while our simulations conserve the total energy. This limitation may be overcome by considering the *convective* K-H instability, which employs inflow/outflow boundaries along the x-direction, as Wu (1986) did in his 2D simulations. However, since one must then use a much larger computational domain to contain the evolving structures, the simulations become substantially more expensive, and it would be currently possible to follow only up to the early stage of the nonlinear development of the instability. A second constraint is due to the periodic nature of the z-boundaries, along the shear plane but perpendicular to the flow direction. From this symmetry the axis of the Cat's Eye is constrained to be perpendicular to the initial flow direction. If the Cat's Eye were allowed to rotate relative to the background flow, it would interact with the flow by crumpling or corrugating. This might lead somewhat different initial nonlinear behaviors in the **WF** and **VWF/WF** cases. Relaxation of that symmetry must await later work. Finally, the reflecting boundaries along the y-direction imposes another limitation of our simulations. However, in 2D, we saw that when activities are limited around the shear boundary as in the **WF** case, the effects of the reflecting boundaries are minimal (see, Frank *et al.* 1996; Malagoli *et al.* 1996). In addition, in the **VWF** case, flow develops into turbulence, so we expect the boundary effects would not be very important to the local properties of the flow.

Thus, we encourage restraint in direct application of our results for interpretation of observational features. Our intent is rather to provide more general physical insights into boundary layer dynamical processes within astrophysical objects subject to the K-H instability, such as jets associated with young stellar objects, accreting binaries, or larger scale flows from active galaxies (*e.g.,* Ferrari *et al.* 1980), strongly sheared flows in the solar corona (*e.g.,* Kopp 1992) and the earth's magnetopause separating the magnetosphere from the solar wind (*e.g.,* Miura 1984). In addition, our findings have several broader implications in astrophysics. The most obvious is that relatively weak magnetic fields may be able to reduce the development of turbulence from the K-H instability and diminish the tendency for mixing and related kinds of transport across slip surfaces. This work, thus, augments earlier suggestions that weak magnetic fields may inhibit turbulent diffusion (*e.g.,* Vainshtein & Rosner 1991).

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TABLE 1
SUMMARY OF INITIAL CONFIGURATIONS

Case[a]	B_x[b]	M_{Ax}[c]	M_{A0}[d]	β[d]	$N_{\rm grid}$
1 (**VWF**)	0.0007	1430	1.24×10^3	1.84×10^6	64^3 (1*l*), 128^3 (1m), 256^3 (1h)
2 (**VWF**)	0.002	500	433	2.25×10^5	64^3 (2*l*), 128^3 (2m)
3 (**VWF**)	0.007	143	124	1.84×10^4	64^3 (3*l*), 128^3 (3m)
4 (**VWF/WF**)	0.02	50	43.3	2.25×10^3	64^3 (4*l*), 128^3 (4m), 256^3 (4h)
5 (**WF**)	0.07	14.3	12.4	184	64^3 (5*l*), 128^3 (5m), 256^3 (5h)
6 (**WF**)	0.2	5	4.33	22.5	64^3 (6*l*), 128^3 (6m)
7 (**SF**)	0.4	2.5	2.17	5.63	64^3 (7*l*), 128^3 (7m)

[a]**VWF**, **WF** and **SF** labels refer to “very weak field”, “weak field” and “strong field” behaviors, respectively, found *for analogous 2D simulations*, as defined in §1. All models used $c_s = 1$, $M_s = U_0/c_s = 1$, $L = 1$, $a = L/25$, $\rho_0 = 1$, and $\gamma = 5/3$.

[b]$B_y = 0$ and $B_z = B_x \tan\theta$ with 30° were used.

[c]$M_{Ax} = U_0\sqrt{\rho_0}/B_x$.

[d]M_{A0} and β are defined from the total initial uniform magnetic field strength, *i.e.*, $B = \sqrt{B_x^2+B_z^2}$.

FIG. 1.— Volume rendering of the strong magnetic field (B) and vorticity magnitude (ω) structures in the medium resolution simulation Case 1m and in the low resolution simulation Case 1l (very weak field, or **VWF** case). Darker regions correspond to higher values and the gray scale was set arbitrarily to highlight structures.

FIG. 2.— Volume rendering of strong magnetic field (B) and vorticity magnitude (ω) structures in the high resolution simulation Case 1h (very weak field, or **VWF** case) at several epochs. Darker regions correspond to higher values and the gray scale was set arbitrarily to highlight structures.

FIG. 3.— Energy evolution in the high resolution simulations Case 1h and 4h and in the medium resolution simulations Case 1m to 4m (**VWF** cases and a transitional case). Shown are the normalized thermal, kinetic, and magnetic energies.

FIG. 4.— Temporal evolution of energy spectra in the high resolution simulation Case 1h (**VWF** case). Shown are the spectra of kinetic energy (E_k), magnetic energy (E_m), and kinetic plus magnetic energy (E_{k+m}). For comparison, solid lines draw $k^{-5/3}$ and k^{-3} power laws. See text for the definition of energy spectra.

FIG. 5.— Temporal evolution of the averaged shear velocity profile in the high resolution simulation Case 1h (**VWF** case).

FIG. 6.— Evolution of some global structure measures in three high resolution simulations (Case 1h, 4h, 5h). Shown are the magnetic curvature radius (L_1), the flow Taylor microscale (L_2), the magnetic Taylor microscale (L_3), and the magnetic intermittency (I). See text for the definitions.

FIG. 7.— Volume renderings of strong magnetic field (B) and vorticity magnitude (ω) structures in the high resolution simulation Case 5h (**WF** case) at several epochs. Darker regions correspond to higher values and the gray scale was set arbitrarily to highlight structures.

FIG. 8.— Temporal evolution of the averaged shear velocity profile in the high resolution simulation Case 5h and in the medium resolution simulations Case 5m to 7m (**WF** cases and strong field case). The bottom panel (e) shows the derivative of the averaged shear velocity around $y = L/2$ at $t = 30$ in Case 5h and 5m and at $t = 20$ in Case 6m and 7m.

FIG. 9.— Energy evolution in the high resolution simulation Case 5h and in the medium resolution simulations Case 5m and 6m (**WF** cases). Shown are the normalized thermal, kinetic, and magnetic energies.

FIG. 10.— Volume renderings of strong magnetic field (B) and vorticity magnitude (ω) structures in the high resolution simulation Case 4h (transitional case) at several epochs. Darker regions correspond to higher values and the gray scale was set arbitrary to highlight structures.

FIG. 11.— Volume rendering showing regions at $t = 8$ for Case 4h (transitional case), where the Alfvén Mach number is less than unity. These regions also trace out rib vortices and are twisted magnetic flux tubes. One such tube structure is identified by a bundle of magnetic field lines are traced in gray.

FIG. 12.— Two iso-surfaces (semi-transparent) highlighting regions of strong to moderate magnetic field strength along with several selected magnetic field lines in the high resolution simulation Case 4h (transitional case) at $t = 40$. Note how one bundle of field lines threads the axis of one of the magnetic strength isosurfaces, which is aligned with the x axis. The other field lines are all chosen to originate on the $z = 0$ face. They meander, but on average are orthogonal to the strong field structure.